The Graphic Demise of FTL

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by

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Catchy title? Actually, I can't kill faster than light (FTL) for real diehards -- that *would* be impossible. What I will try to do here is to show *how* FTL systems create paradoxes and then briefly comment on why some standard SF conventions don't really get around this.

The Speed of Light is Constant

The main problem with FTL comes from experiments that show that the speed of light always measures the same regardless of how fast one is moving with respect to the light source. Figure 1 is representative of these experiments. In it, a pulse of light encounters two speed of light measuring apparati, one moving away and one toward the pulse of light. Despite the velocity difference, the light only gets bluer (approaching) or redder (receding) by Doppler shift. The measured speed *stays the same*.

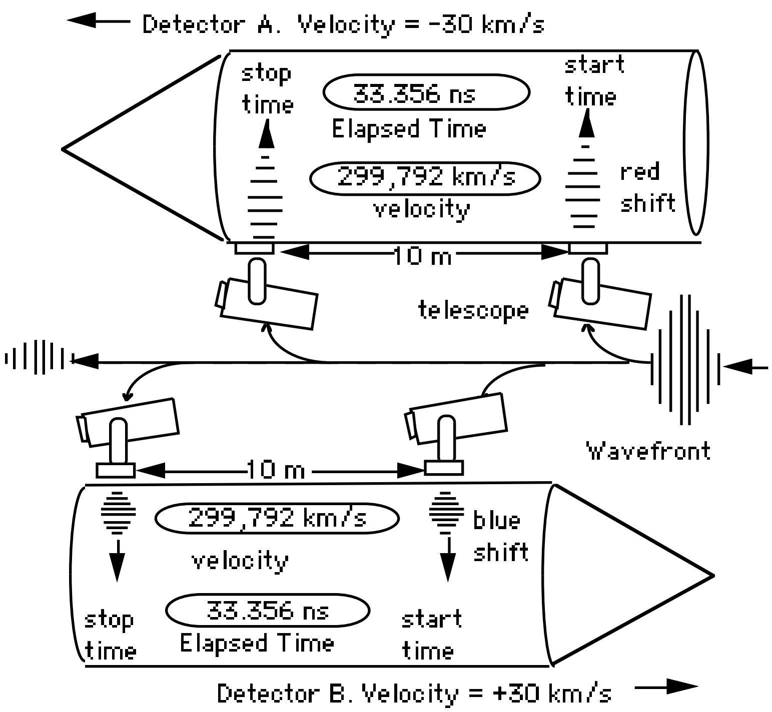


Figure 1. Spacecraft at different velocities measuring the velocity of the same wavefront.

Actually, the universe would look really weird if this weren't true. For instance, consider how orbiting binary stars would look if light from the approaching star got here before light from the receding star. Binary stars, however, aren't distorted that way -- we see them go around each other normally.

But a constant speed of light still seems strange. It's as if a pitcher (or bowler, UK friends) throws a ball at you at 40 m/s, you run away from it at 10 m/s, and it *still* hits you at 40 m/s. Life may be like that, you say, but physics shouldn't be.

The Lorentz Transformation

Fortunately, sanity can be preserved if both the time and distance scales used by someone moving with respect to you appear to contract so the speed (distance/time) comes out the same.

The equations that describe this are called the Lorentz transformation. (see the box at the end). A Dutch physicist, Hendrik Lorentz was the first person to observe that distance contracts *and* mass increases asymptotically as velocity approaches the speed of light.

Einstein's contribution was the concept of relativity. Loosely stated, the laws of physics as measured by any observer work the same regardless of his or her motion with respect to any other part of the universe. This is very fortunate because every part of the universe is moving with respect to every other part and if the laws of physics changed with motion, none of the chemistry that holds us together would work the way it needs to work to keep us alive!

Special relativity deals with constant motions and is built of simple, easy steps that anyone can understand. Except for the results of those speed of light measurements, nothing needs to be taken on faith. All the math one needs for graphical special relativity is a little plug-the-numbers-in-the-formula high school algebra. Anyone who can figure out how long it takes to go 100 miles at fifty miles per hour can handle this.

Picturing Space and Time

We start by looking at ordinary space and time, as on an automobile trip. In figure 2, this trip is represented by slices of space that show where the car is as time progresses. Since we can't see time, we use another dimension, in this case "up," to represent time. The tree stays in the same place in space and "moves" up in time. Distance is horizontal. In each slice of time, the car moves to the right.

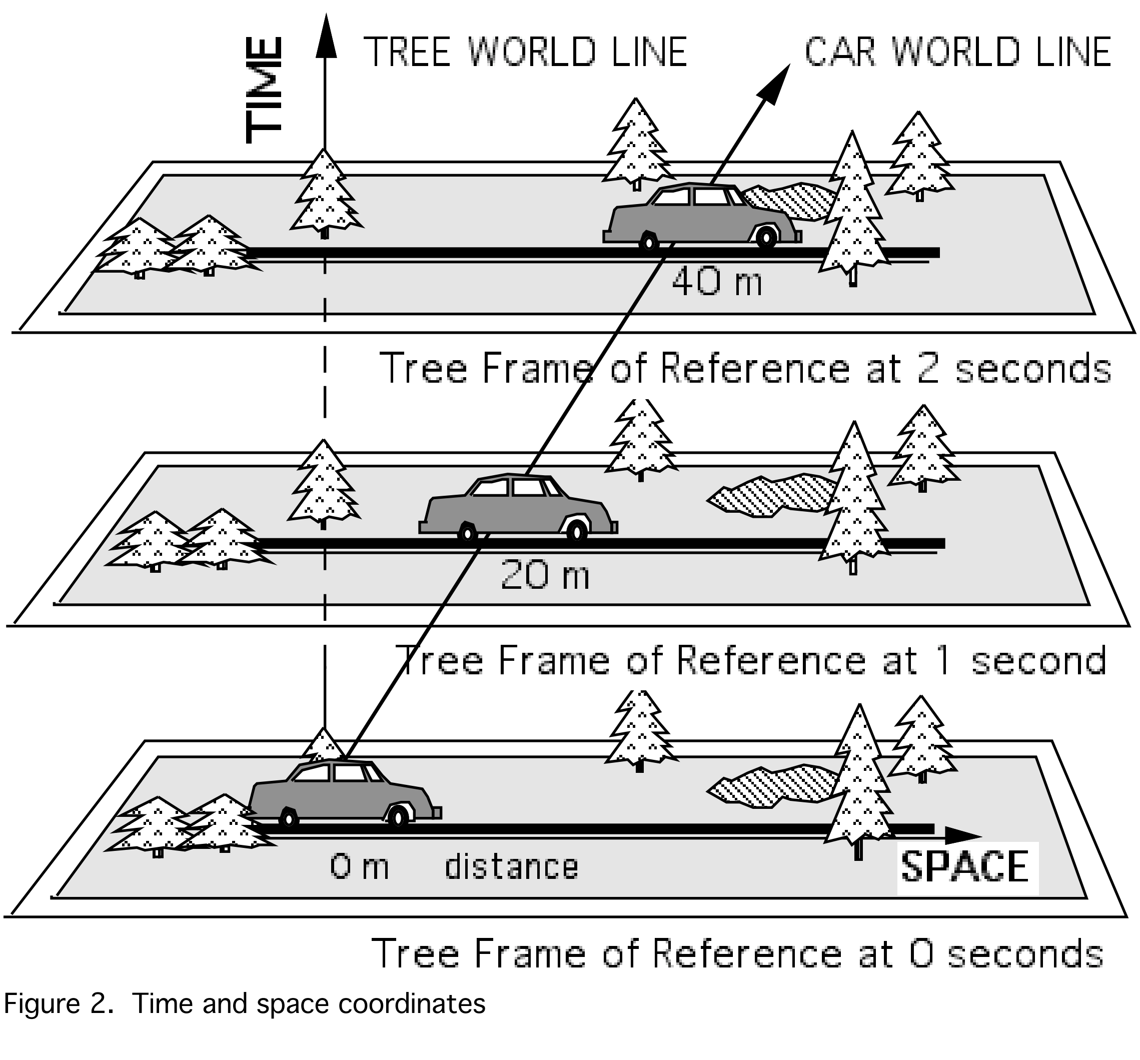


Figure 2. Time and space coordinates

Height and depth can be inferred from perspective, but as long as we introduce no accelerations in those directions, we can neglect them and work with just distance and time. Points on this set of pictures are called "events," as they each have a particular time and place. The path of an object through space and time is a continuous string of events called its "worldline."

Objects that are stationary with respect to each other define a "frame of reference" and they have worldlines that are parallel. Objects moving with respect to that frame of reference, such as the car, have world lines that are slanted compared to the others.

The convention in physics is to call the frame of reference drawn with the vertical time axis the "laboratory," or "lab," frame of reference and the squashed frame of reference the "proper frame." (One can remember the latter by thinking of it as the proper, i.e. correct, frame to measure time and distance as viewed by a body moving with respect to the "lab frame.")

But, since any frame can be considered stationary, it may be less confusing to name frames in a way that doesn't change with the viewpoint. For star travel, we'll use "sidereal" (meaning "of the stars") frame for the Earth, Sol and (relatively) fixed destination stars and "starship frame" for the starship's frame.

Now we have to get graphic. In figure 3, a starship takes two lightyears up in time to go one lightyear across in distance, for a world line with a slope of 2. The solid vertical line again measures time, "t," now in years, and the solid horizontal line measures distance, "x," in lightyears.

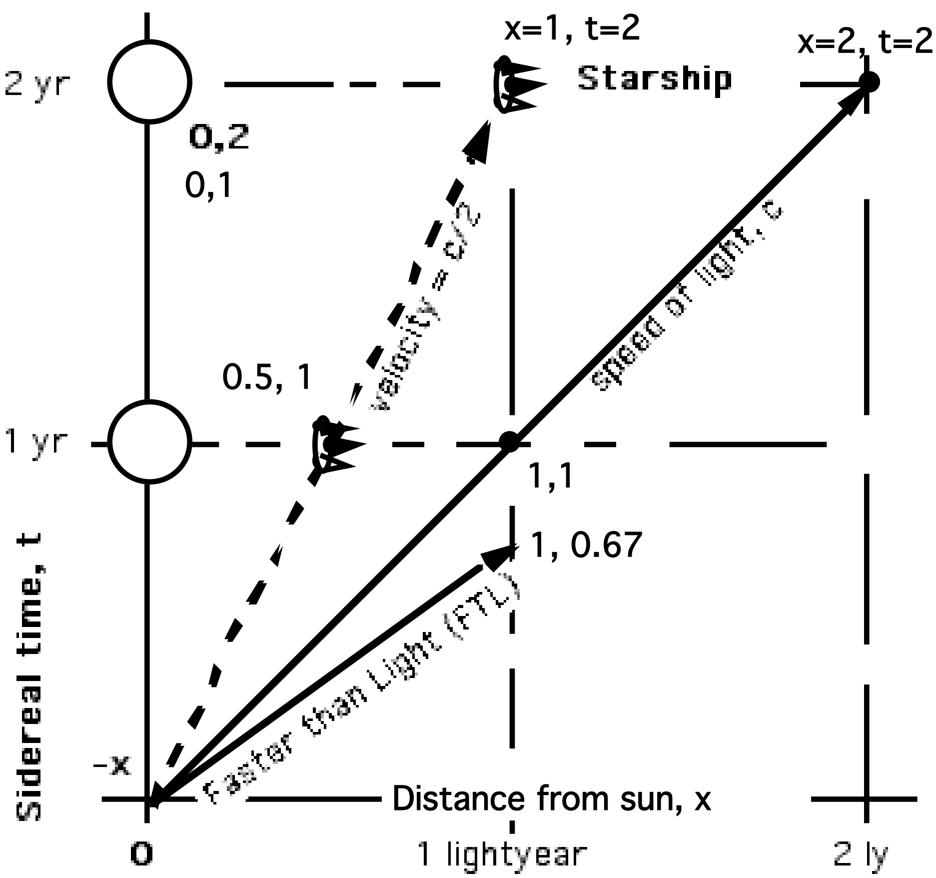


Figure 3. Spacecraft in Solar frame coordinates

Events have x and t coordinates, usually written with x first and separated by a comma. For instance, the starship is shown at (1, 2) which means it's at an event one light year and two years from the origin. Light from the origin (0, 0) traces a worldline with a velocity “c” of 1 (or -1) lightyear per year, and thus forms worldlines with a slope of one lightyear per year. Things that move slower than light trace worldlines with a higher slope; they take more time for the same distance. If something could go faster than light, it would trace a world-line with a lower slope than c. Got it?

Now, how do things look from the starship? Note that the starship is not moving in its own frame of reference. This seems trivial but it's critical because it helps us define the starship's reference frame. The starship is always at zero distance in its own frame of reference, so the new time axis is simply where the starship is, was and will be--its worldline.

But how do we measure distance from the starship? The intuitive, everyday assumption would be to leave the distance axis where it is, and for ordinary velocities, like those of our car, this works just fine as long as you don't look too closely. But the speed of light wouldn't (and didn't) come out right if measured precisely.

To keep c constant as measured on the starship, we have to graphically mutilate our right-angle Cartesian instincts and tilt the starship's distance axis up so it forms a narrow "lazy v" with the time axis, instead of the right angle made by most coordinate systems.

In figure 4, the dashed lines define the starship's squashed (from our point of view) "frame of reference." The starship's distance axis is labeled *x'* (say "x prime") and its time axis is labeled *t'* ("t prime"). The usual notation is for the primed coordinates to go with the proper frame (*x's*, *t's* etc.).

Lines where distance from the starship is constant are parallel to the *x'* axis, and lines where time is the same as on the starship are parallel to the starship's worldline. This graph, showing both coordinate systems, is known as a "Minkowski diagram." In both coordinate systems the speed of light world line must go *right down the middle*, equidistant between the space and time axes. This is the way it has to be to keep c constant. Geometrically, the *slope* of the distance axis must always be the reciprocal of the slope of the time axis/velocity line (2 years/lightyear for time, 1/2 year per lightyear for distance).

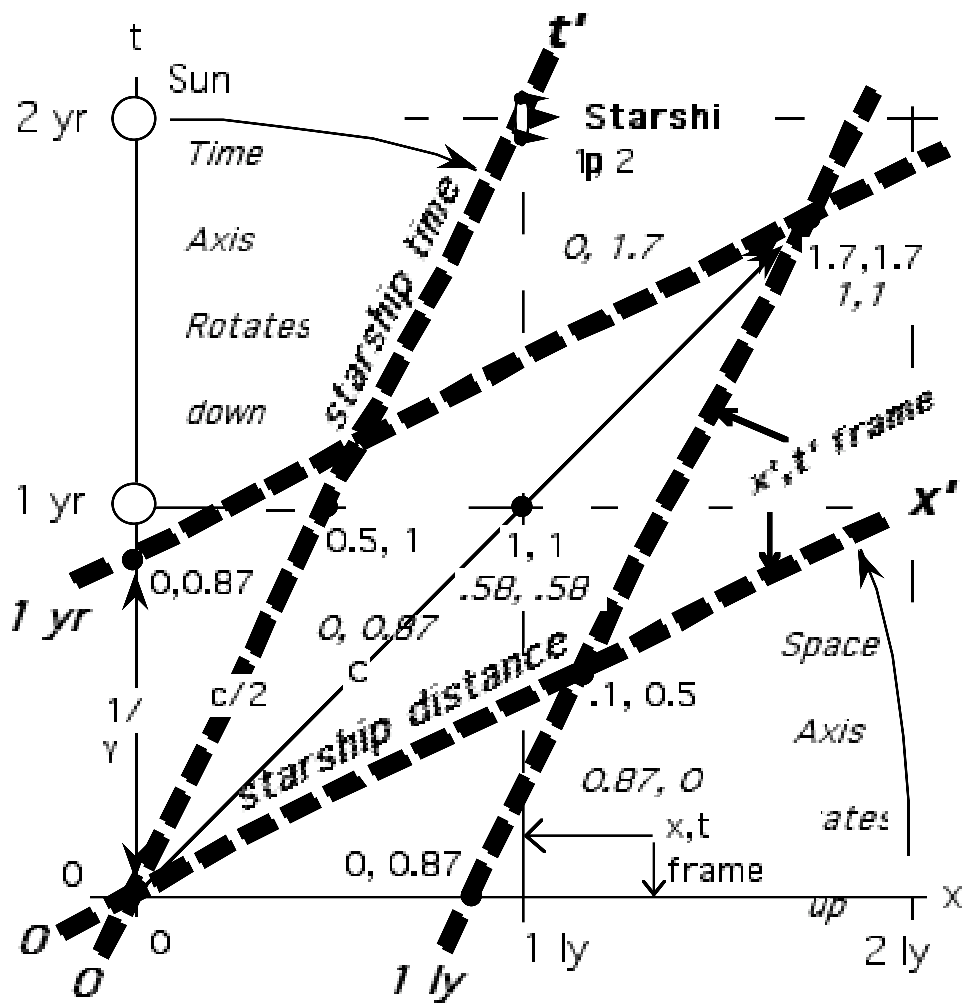


Figure 4. Starship frame of reference overlaid on Sun's frame of reference.

The Lorentz transformation tells us that time and distance in the starship's frame appear *contracted* by a factor of g, so the new starship frame's *1 year* and *1 lightyear* grid lines will intercept (cross) the sidereal frame axes at 1/g years and 1/g lightyears respectively. At half the speed of light, g = 1.15 and 1/g = 0.87. (See the box following this article). With the slope of the new coordinate lines (they are parallel to the *x'* and *t'* axes), we can draw them through their intercept points.

The new coordinate grid looks like the cross section of a set of half-folded bottle packing dividers, but, despite all the downsizing, it can still do its job. That job is to assign unique coordinates to all events. Every event has a set of co- ordinates in the sun's frame of reference, (x, t) *and* in the starship's (*x', t'*).

There are, of course, an infinite number of reference frames for the infinite number of relative velocities between 0 and c. Note that we can overlay as many arbitrary reference frames over the same events as we want without changing anything -- only the events themselves are real.

Also note that the new time and distance lines cross each other at *1,1* or 1.7, 1.7 on the speed of light line. The distance light travels per unit time, its velocity is *1/1* or 1.7/1.7: the same in both frames. Having the speed of light the same in all such frames is what satisfies the experimental results.

What's Good for the Goose...

Special relativity lore states that:

(1) All inertial frames of reference are equivalent. There is no special frame, but....

(2) A twin who visits another frame of reference and returns will be younger--a result that appears special.

The apparent contradiction between these statements bothered me for years. How could both be true? This seeming paradox cast a cloud of uncertainty over special relativity, and hopes for FTL hid among these clouds.

But dealing with graphs instead of statements helped resolve this. Note that in figure 4, the x = 1 ly and t = 1 yr gridlines also cross the *x'* and *t'* grid lines at 0.87. Both the sidereal frame *and the starship frame* of reference appear contracted from the other frame.

Consider, as Einstein did, passing twin relativistic trains with clocks in their engines and cabooses, as in figure 5. Clocks in both engines read 0 when they cross. The figure is drawn in the frame of reference of Train B (black), Which moves vertically up in time as Train A (gray) passes it.

Now, what elapsed times do the clocks in the cabooses show as they cross?

The same times, both about 36.6 μs.

Indeed, *every* car's clocks will match as they pass. So, how can you get an unequal result, like one twin being younger after a space voyage? What happened to the "twin paradox?"

Note how space in one frame of reference is tilted with respect to time in the other. The real Train A must be measured in this "proper frame," where all its parts exist simultaneously (i.e. along a line of constant *t'*) and *this* train slants down (back in time) with respect to the other train's x axis. The clock in the caboose of right-moving Train A read 0 in the *past* of its twin Train B. What observers strung out along Train B's track would measure is a shorter "Apparent Train A" (white) with its cars all at different times.

If a twin (or a clock reading--to avoid irrelevant complications about acceleration) gets on Train A at 0 sec., its elapsed time will indeed be less when it goes back to Train B--but the same would be true of a twin that left Train A, stayed on Train B for a while and then returned to Train A.

So, what's good for the goose is good for the gander. It makes no difference whether you draw the graph from the Sun's point of view (with t vertical) or from a starship's point of view (with *t'* vertical), as in the left half of figure 6.

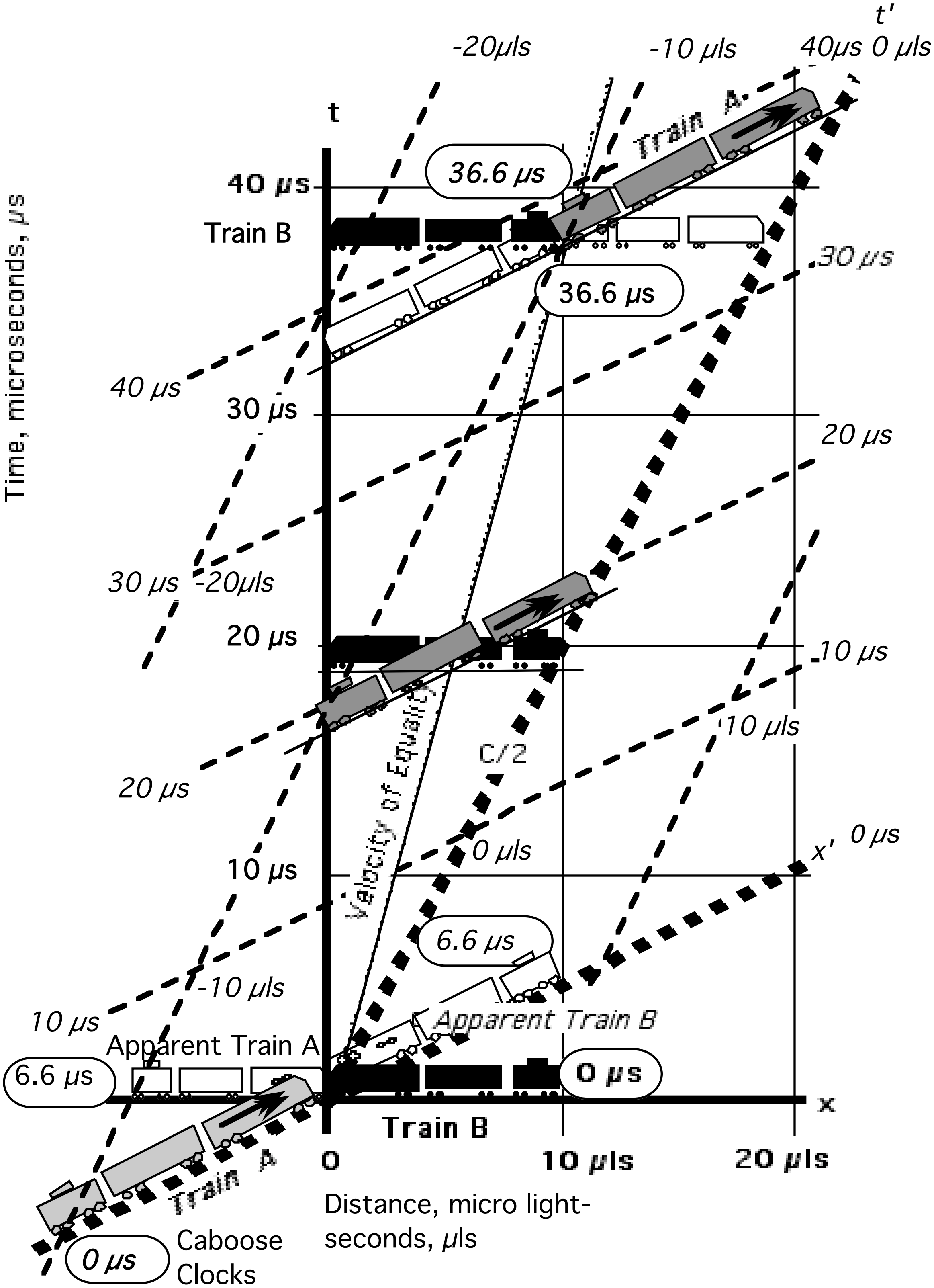


Figure 5. Passing trains

We can even represent space time from the point of view of a frame of reference with a velocity halfway between the starship and the stars, as in figure 6.

No matter what the graphic representation, all events still have the same coordinates and all intervals (such as a round trip to a Centauri) will be the same. Asymmetries in round trip elapsed times come from the choice of a rendezvous point. Choose a symmetrical rendezvous (at the “velocity of equality” in figure 5, for example) and the elapsed times will be the same.

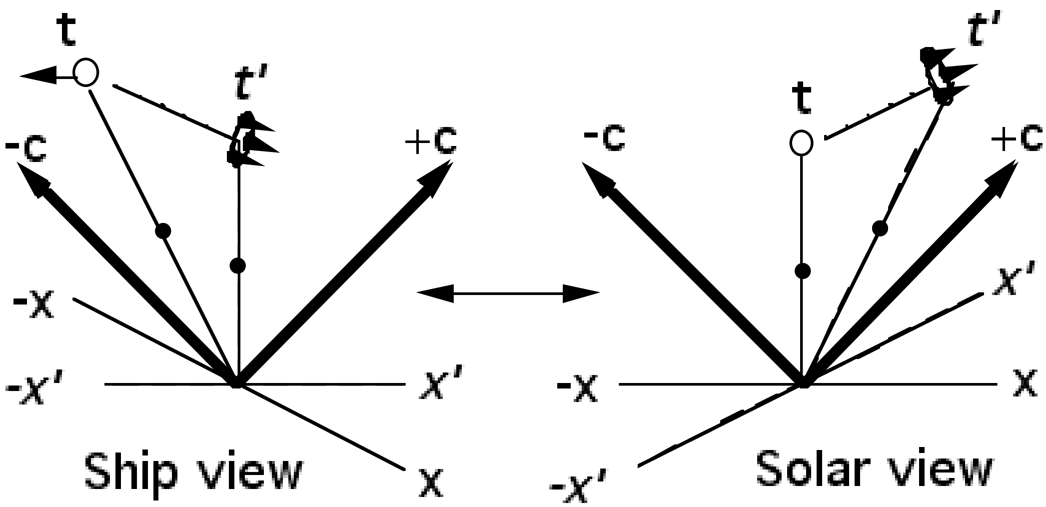


Figure 6. Lorentz transformation symmetry.

Ordinary space geometry can illustrate this symmetry between frames of reference. Suppose we are in the meteor crater in Arizona looking up at, say, Clavius crater on the Moon. Clavius is far from the Moon's equator and appears foreshortened into an oval. An observer at Clavius crater would see the Arizona meteor crater, north of our equator, looking somewhat oval as well.

But both observers would see their own craters as roughly circular.

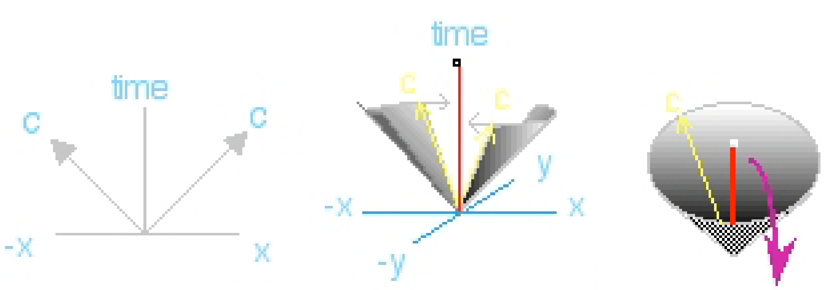
Likewise, within any spacetime frame of reference, everything local looks perfectly normal and clocks don't run slow or backward. It's the *rest* of the universe that looks warped.

This works. People have been using it with subatomic particles for years and years and getting results that conform to their predictions to the umpteenth decimal point. There is nothing, contradictory, strange, "theoretical," or otherwise flaky about this. It is the way the macroscopic universe is just as much as we know that the world is a globe.

More Dimensions

Now, what happened to those other dimensions of height and depth? Essentially nothing. If we add another spatial direction "y," for depth, instead of a "light V" we get a "light cone."

Look into the light cone, and you look back in time along your own worldline (your own time axis). The space axes of the other objects, x and *x'* for the starship, slant to keep the cone walls at a constant slope of "c". The *y'* axis moves out to the right, but stays vertical. Tilting and contraction occur only in the direction of relative motion.



FTL is Time Travel

That's the background, now what about FTL? What lies behind the light cone? *There,* and not with the twins, be the dragons of paradox. In figure 7, a starship is headed to the right and the stars to the left.

Consider a message sent from event "A" into some FTL device, say a wormhole, and exiting to the starship at event "B" as in figure 7.

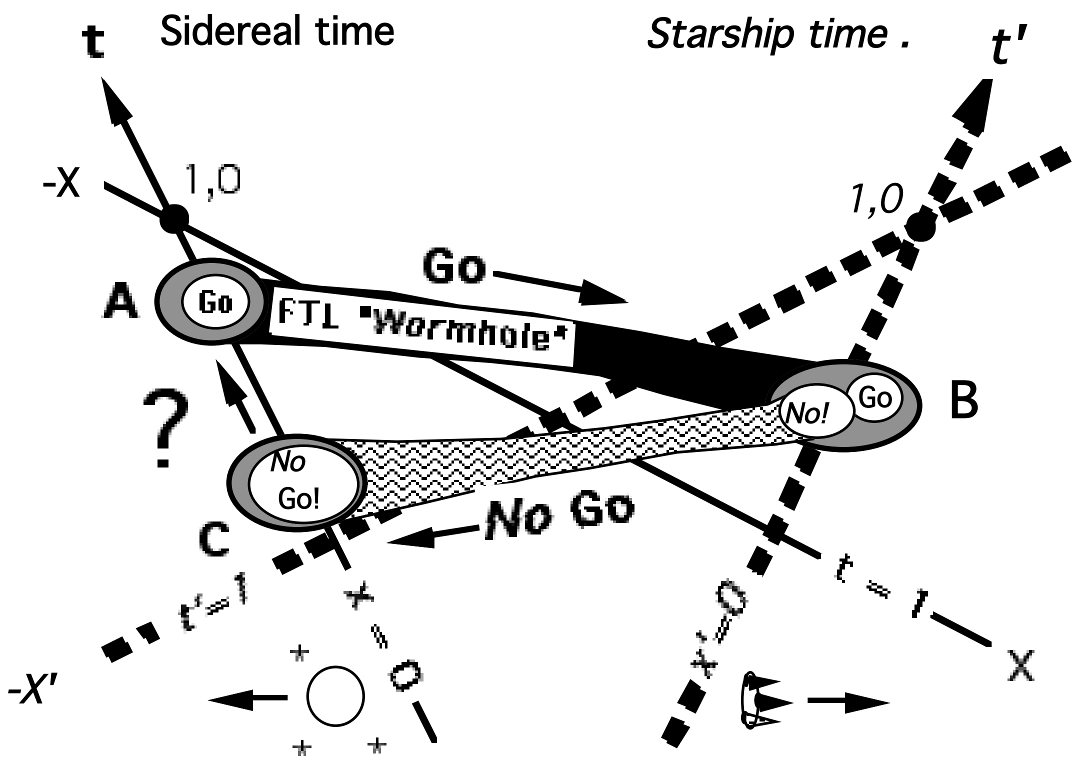


Figure 7. FTL Paradox Generator

So far, okay. The message went FTL, but still forward in time in the sidereal frame (A is below the t = 1 yr. line, B is above it). But then at event B the starship replies through its FTL device, saying "don't send that message, or all reality will be destroyed." This anti-message arrives at the solar system as event C, having gone FTL, though still forward in time *in the Star-ship's frame* (B is below the t*'= 1 yr* line and C is above it). But event C comes *before* A in all frames, so fearing destruction, no message is sent at A.

Which means the anti-message is never sent. Which means A's message is sent. What this all means is that if FTL exists, one can demand that two mutually exclusive histories exist in the same universe.

Is there any way out of this? In his novel *Time for The Stars*, Robert A. Heinlein had twins communicate by instantaneous telepathy, forcing Heinlein to choose one of his twins to live faster to allow a real-time Doppler-shifted conversation. But in a true Lorentz transformation, one twin's answers would arrive before questions, creating utter paradoxical confusion.

For instantaneous communications to work, all events would have to be ordered on one preferred time axis (though rates could vary). No distance axes could slant back in some other frame's time. But when you try to make c constant in such a system--oops!

Some very bright people keep trying this in some very complex ways, but without recognized success. In graphic terms, that lower distance axis still has to rotate upwards, however ugly the result. These graphs really just say what the experiments and the says--that c (in vacuum) is always constant, period.

Dealing With Paradox

Yes, there are ways around causal paradoxes, kind of.

One can assume everything that happens is rigged in advance so that, for instance, the message: "Don't send the message" just doesn't get sent--or it just isn't obeyed. Many time travel stories work that way, notably Vonnegut's *Slaughterhouse 5*. But that means nothing we do ever makes any difference because everything is already locked into a crystalline eternity-- "...and so it goes," as Vonnegut's Billy Pilgrim said.

I'd rather take twenty years to get to Tau Ceti, thank you.

Another way of dealing with paradoxes is to assume that travel backward in time shifts you into a different universe that's the same up to that point, but splits off into a different history then.

Well, maybe. But that doesn't address how to go somewhere FTL in *this* universe, and so is outside the scope of this article.

An important point to remember is that *it doesn't matter how something tries to go FTL*. You can go through the wormhole, around the cosmic string, into the alternate universe, back through the chrono-simplastic infidibulum and call home on the ansible--and none of these things eliminates the paradox problem. The only thing that matters is *when* and *where* the the signal or thing arrives. If it arrives before light from its source could arrive, then you've got a potential paradox-maker. Trying to avoid this consequence by going "out of the universe" or "going through as space warp" is like trying to avoid the consequences of the international dateline by digging a tunnel under it.

No Barriers

It is sometimes said that the "barrier" to FTL is that as v approaches c, the denominator of g goes to zero (see box), and g goes to infinity. Relative mass-energy increases with g, and the force and energy needed to accelerate near infinite mass must be near infinite too.

But, they say, that's only a problem when v is very close to c. If B is greater than c, the Lorentz factor is not infinite but merely imaginary (multiples of $\sqrt{-1}$ are "imaginary" numbers). So going around c in some other dimension, or inventing tachyons that always move faster than light could (imaginarily, anyway) avoid division by zero.

But that still leaves time paradoxes.

Now a ray of hope. What do we mean by velocity? When we take a journey we divide the distance on the road, or the map, by the time it takes on our watch in the car. By this measure, there is no limit to velocity.

Note how the starship frame gets stretched out as it gets squashed? The starship itself *sees no light speed barrier*--it can reach any place in the Sun's frame in as little on-board time as its propulsion system allows: this was the point of Anderson's "Tau Zero." The starship just moves faster and faster as things ahead of it get closer and closer and bluer and bluer.

While the classic “Bussard ramjet” propulsion system is unlikely to work, recent research shows that starships can be pushed by beams-- fountains of photons or relativistic pellets--generated by the originating planetary system. The kinetic of a starship traveling near c exceeds to the mass-energy of the starship (imagine the starship being half antimatter) -- but self-replicating machines can build the solar power stations needed to capture the necessary energy in a few decades. Even a billionth of the sun’s output is enough to push a starship’s velocity to very impressive Lorentz factors.

Anyway, since there is no barrier, there is nothing a starship can do to "break through" the non-existent barrier. The problem is a time coordinate at the distant place in the sidereal frame when the starship gets there, and that is utterly beyond the starship's control.

Finally

The important point to remember from all this is that from the above, by Lorentz and Einstein, any FTL travel *is* time travel. So FTL should not be considered a "mere" technology problem like the sound barrier, biological immortality, brain-computer interfaces, artificial intelligence, etc. FTL means paradoxes, predestination, or a massive rejection of a century of physical measurements relating to the relative nature of macroscopic space and time -- all of which seem very unlikely to this writer.

Meanwhile, the Lorentz transformation is how our universe works--and perhaps the only way any universe can work. That's the graphic reality of it.

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Box: For the convenience of those who want to do calculations, the Lorentz transformation equations are:

$$B = u/c$$
$$\gamma = 1/\sqrt{(1-B^2)}$$
$$x' = \gamma\ (x-B\ c\ t)$$
$$x = \gamma\ (x'+B\ c\ t')$$
$$t' = \gamma\ (t-B\ x/c)$$
$$t = \gamma\ (t'+B\ x/c)$$
$$v' = (u-v)/(1-uv/c^2)$$

where u is the relative velocity between the primed (x’,t’) and unprimed (x, t) frames of reference, c is the speed of light, x and x’ are distances, t and t’ are times, γ (gamma) is the Lorentz factor, v is a velocity measured in the unprimed frame of reference, and v' is the same velocity measured in the primed frame of reference. This is simplified if one uses distance in lightyears and time in years; that way, c =1 and so can be removed.

2006 addendum

$$v' = (u-v)/(1-uv/c^2)$$

This last equation above lets us go back to the baseball thrown by the receding pitcher. You are running away from the pitcher with velocity "u" and the pitcher throws the ball at velocity "v" relative to him. v' is the velocity of the ball with respect to you. A negative number means the distance between you and the ball is decreasing; i.e. the ball is going to hit you. At ordinary relative velocities, c^2 is so large that uv/c^2 is tiny and can be neglected for most purposes. Let's say you run at 9 m/s and the pitcher throws the ball at 36 m/s. Well, uv/c^2 is about 1.6e-15 and you would need an instrument able to measure a velocity difference of 4.3e-9 m/s. (Scarily, a gigahertz interferometer might be able to do just that!)

For an effect that has some practical significance, though, the pitcher must throw with relativistic velocity. Lets say what's being thrown is a particle beam at 0.2 c and what's running away is a spaceship moving at 0.1 c. The naive assumption would be that the particle beam would have a velocity of 0.1 c in approach relative to the spacecraft, but our equation tells us that the velocity of the particle approaching the spacecraft is actually about 2% more than that.

The closer v gets to c, the more the baseball/particle looks like a photon. If the ship has reached 0.67 c and the particle beam is chasing it at 0.72 c, the inbound velocity is about .1 c, instead of the 0.05 c difference ship and the beams velocity with respect to the beam throwers. If v gets really near c, the inbound velocity approaches c, regardless of the velocity of the fleeing ship. See what happens to our equation when we replace v with c:

$$v' = c^2(u-c)/(c^2-uc)$$

$$v' = c(u-c)/(c-u) = -c(c-u)/(c-u) = -c$$

The velocity u disappears entirely. It does not matter how fast you run, anything approaching at the speed of light is going to hit you...at the speed of light.

Derivation of the Lorentz transformation.

The Lorentz factor and the transformation equations follow directly from the experimental fact that c(in vacuum) is the same in all frames of reference. For simplicity, we'll confine this to one spacial dimension and time. Let a photon travel between two arbitrary events, 1 and 2, with coordinates (xa1, ta1) and (xa2, ta2) in frame "a" and coordinates (xb1, tb1) and (xb2, tb2) in frame "b." The relative velocity between the frames is "v". The velocity of the photon "c" between events is then: c = (xa2 - xa1)/(ta2 - ta1) in the a frame and c = (xa2 - xa1)/(ta2 - ta1) in the b frame.

Without any loss of generality, we can set xa1, ta1, xb1 and xt1 to zero and drop the subscripts 1 and 2, understanding that ta and tb now stand for the times of flight in their respective reference frames, and xa and xb stand for the distance covered. So:

(1) c = xa/ta = xb/tb

Naively, we would expect the position xb in the "a" frame and xa in the "b" frame to be given by:

(2a) xb = xa + v*ta

(2b) xa = xb – v*tb.

It's zero to start with and the fames are separating by velocity v, so distance is rate times time, right?
If I divide each of these by the time coordinate, I get:

xb/ta = xa/ta +v and xa/tb = xb/tb - v.

Then using (1) above I get:

xb/ta = c + v and xa/tb = c - v

I can multiply these two equations and get

(xb/ta)(xa/tb) = $c^2 - v^2$

Rearranging the left side, I get

(xb/tb)(xa/ta) = $c^2 - v^2$

But again from equation (1), (xb/tb) = (xa/ta) = c, so

(3) $c^2 = c^2 - v^2$

This can't be true unless v = zero, however. What to do? The genius of Nobel quality mathematical physicists is that they can think outside the box. Let us, they say, modify equations (2a) and (2b) with a factor T so that:

(4a) T*xb = xa + v*ta and

(4b) T*xa = xb – v*tb

Note that the *same* factor modifies both 4a and 4b. Symmetry is preserved. Now, if we go through the above procedure (divide by the time coordinates, multiply equations and perform the substitutions) we get, instead of (3), the following:

$T^2c^2 = c^2 - v^2$

Which makes everything work out as long as:

(5) T = sqrt($c^2 - v^2$)/c^2

And there you have it! "T" is the "tau" of Poul Anderson's famous *Tau Zero,* which happens if v = c. To get the Lorentz transformation equations above, replace T with γ = 1/T, ta with t, tb with t' and v/c with B. Multiply by γ to give you the distance transformations above, then solve for t' and t to give you the time transforms.